UNIT - IV

DEFUZZIFICATION

- Consider fuzzy set $\underset{\sim}{A}$ defined a lambda-cut set for set $\underset{\sim}{A}$ denoted as A_λ (crisp set) where the value of λ ranges|lies b/w

$$0 \leq \lambda \leq 1$$

- The set A_λ is a crisp set called the lambda cut or alpha cut set of the fuzzy set $\underset{\sim}{A}$ where

Forall

$$A_\lambda = \{ x \mid \mu_{\underset{\sim}{A}}(x) \geq \lambda \}$$

$\lambda \rightarrow$ values are membership value

- It is a crisp set derived from its fuzzy set $\underset{\sim}{A}$
- Any fuzzy set $\underset{\sim}{A}$ can be transformed into an infinite number of alpha cut set or lambda cut set because there are infinite no. of values λ on the interval $[0,1]$
- Any element $x \in A_\lambda$ belong to $\underset{\sim}{A}$

$x \in A_\lambda \rightarrow \underset{\sim}{A}$ with a grade of membership i.e greater than or equal to the value of λ

- Let us consider an eg as shown below

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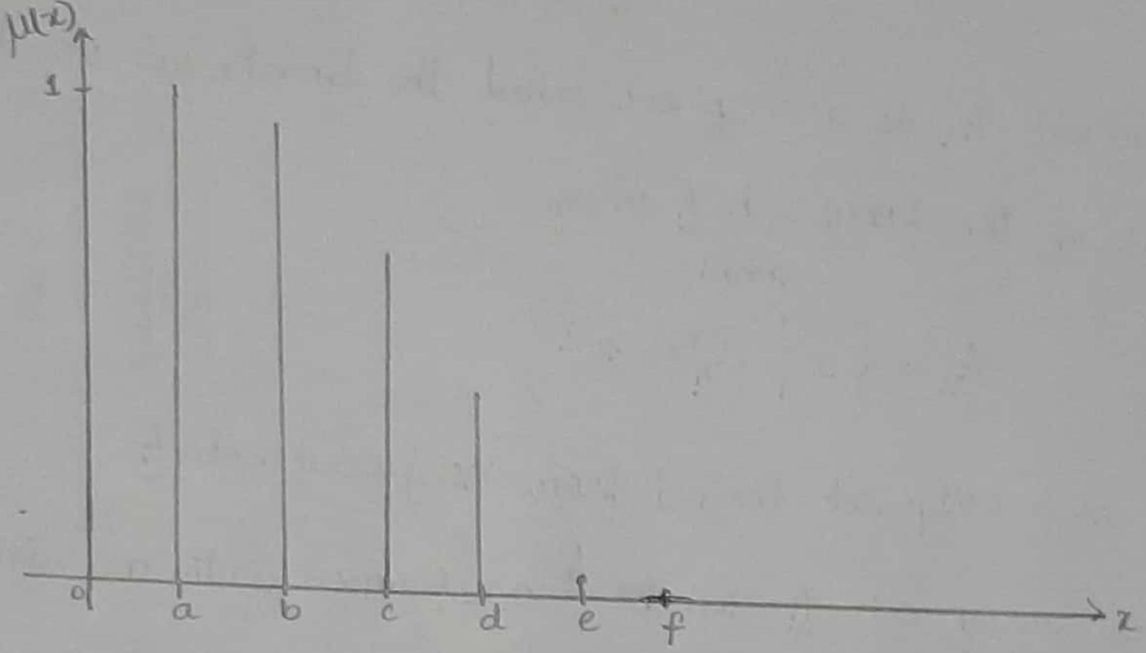
Example:

Let us consider the discrete fuzzy set defined on the universe X $X = \{a, b, c, d, e, f\}$

- Let the fuzzy set $\underset{\sim}{A} = \left\{ \frac{1}{a} + \frac{0.9}{b} + \frac{0.6}{c} + \frac{0.3}{d} + \frac{0.01}{e} + \frac{0}{f} \right\}$

- The plot of fuzzy set $\underset{\sim}{A}$



- This fuzzy set $\underset{\sim}{A}$ can be reduced into several alpha cut sets which are all crisp in nature.

- The alpha (or lambda) cut sets for the value of $\lambda = 1, 0.9, 0.6, 0.3, 0.01, 0$

$= 1, 0.9, 0.6, 0.3, 0^+ \text{ \& } 0$

null mem

slightly more than null

greater than this is e

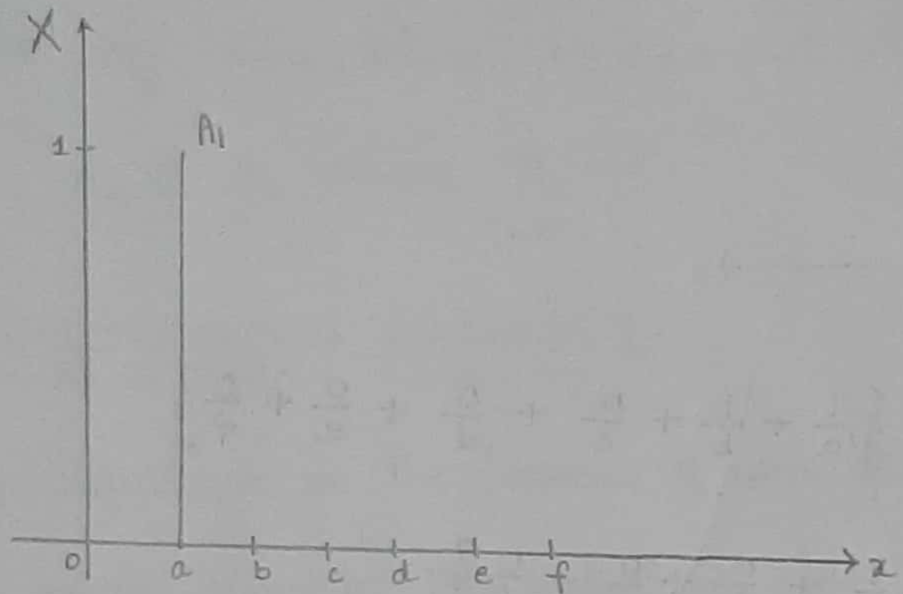
$A_1 = \{a\}$ $\quad A_{0.9} = \{a, b\}$ $\quad \boxed{\mu_{\underset{\sim}{A}}(x) \geq \lambda}$

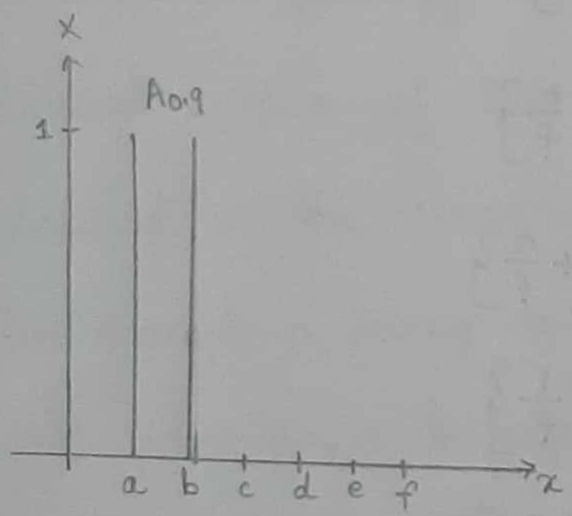
$A_{0.6} = \{a, b, c\}$

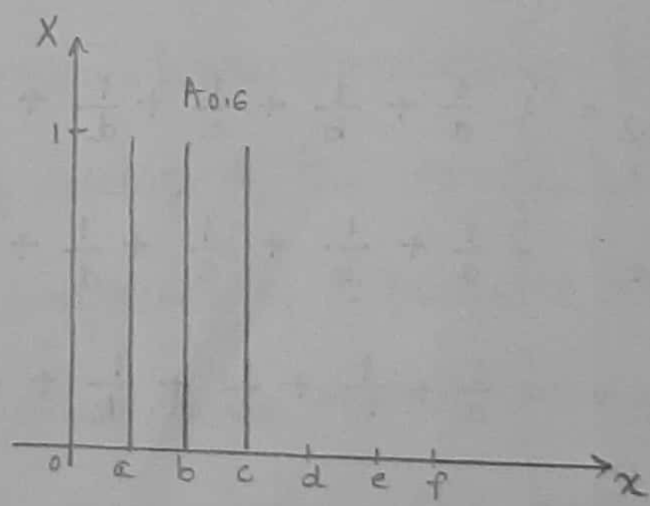
$A_{0.3} = \{a, b, c, d\}$ $\quad A_{0^+} = \{a, b, c, d, e\}$

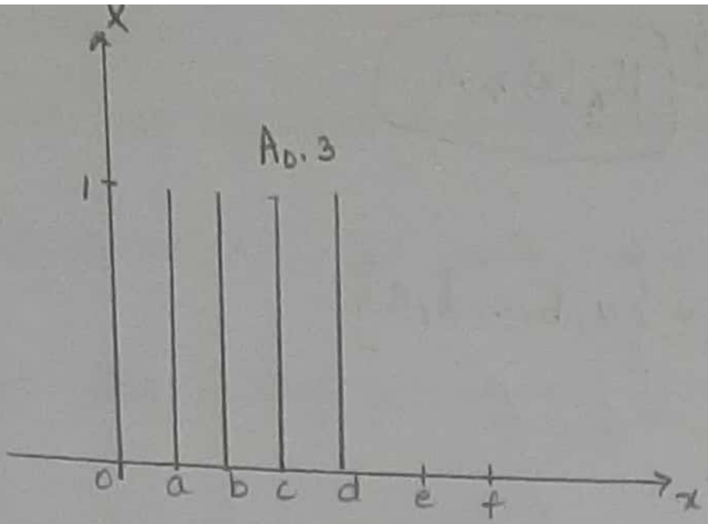
$A_0 = \{a, b, c, d, e, f\} = X$

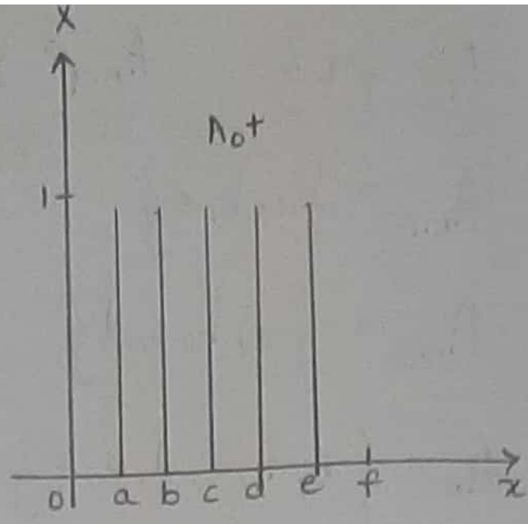
→ lambda cut set, schematic dig or representation

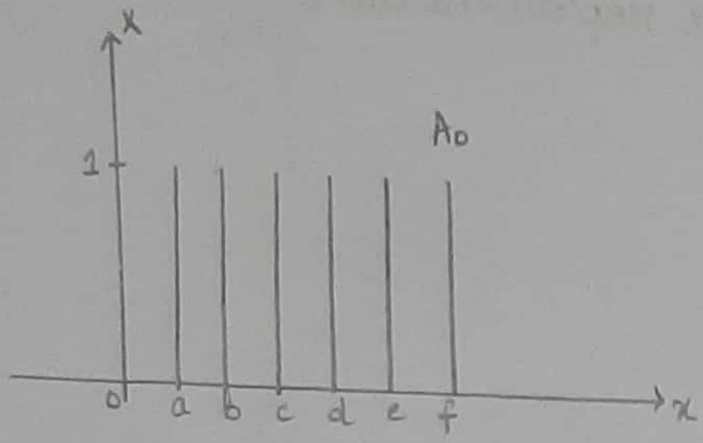












$\cancel{A_1 = \left\{\frac{1}{a}\right\}}$ $\quad A_{0.9} = \left\{\frac{1}{a} + \frac{1}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e} + \frac{0}{f}\right\}$

$$A_{0.6} = \left\{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{0}{d} + \frac{0}{e} + \frac{0}{f}\right\}$$

$$A_{0.3} = \left\{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{0}{e} + \frac{0}{f}\right\}$$

$$A_{0+} = \left\{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} + \frac{0}{f}\right\}$$

$$A_{0} = \left\{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} + \frac{1}{f}\right\}$$

$$A_{1} = \left\{\frac{1}{a} + \frac{0}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e} + \frac{0}{f}\right\}$$

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Properties of lambda cut set :

- lambda cut sets have the following 4 special properties they are:

1. $(\underset{\sim}{A} \cup \underset{\sim}{B})_\lambda = A_\lambda \cup B_\lambda$

2. $(\underset{\sim}{A} \cap \underset{\sim}{B})_\lambda = A_\lambda \cap B_\lambda$

3. $(\overline{\underset{\sim}{A}})_\lambda \neq \bar{A}_\lambda$ except for the value of $\lambda = 0.5$.

4. For any $\lambda \leq \alpha$, where $0 \leq \alpha \leq 1$, it is true that

$$A_\alpha \subseteq A_\lambda \text{ where } \underset{(\lambda=0)}{A_0} = X$$

DEFUZZIFICATION METHODS :

- Defuzzification is the process of converting a fuzzy quantity to a precise quantity. The output of fuzzy process can be logical union of two or more fuzzy membership funct defined on the universe of discourse of the o/p variable.
- For example a fuzzy o/p is comprised of 2 parts i.e. the first part $\underset{\sim}{C_1}$, a trapezoidal shape & the second part $\underset{\sim}{C_2}$ is a triangular shape. The union of these two m.s function $\underset{\sim}{C} = \underset{\sim}{C_1} \cup \underset{\sim}{C_2}$ involves the max operator which is graphically

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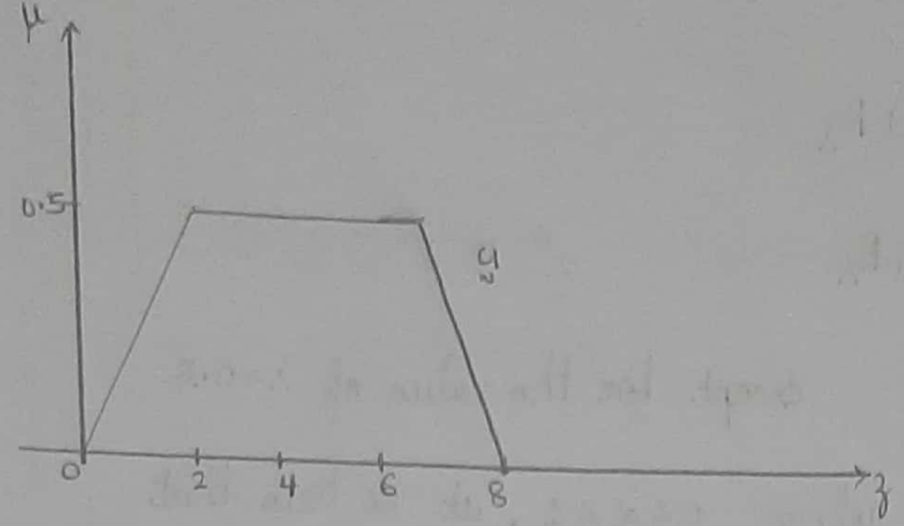
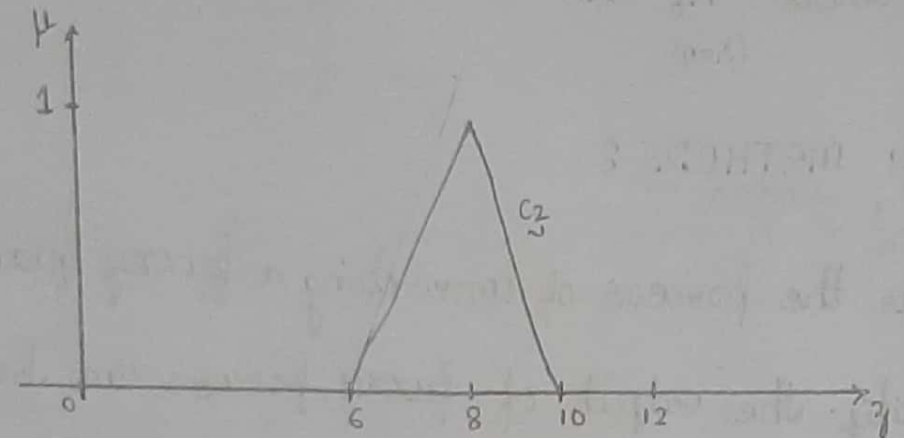
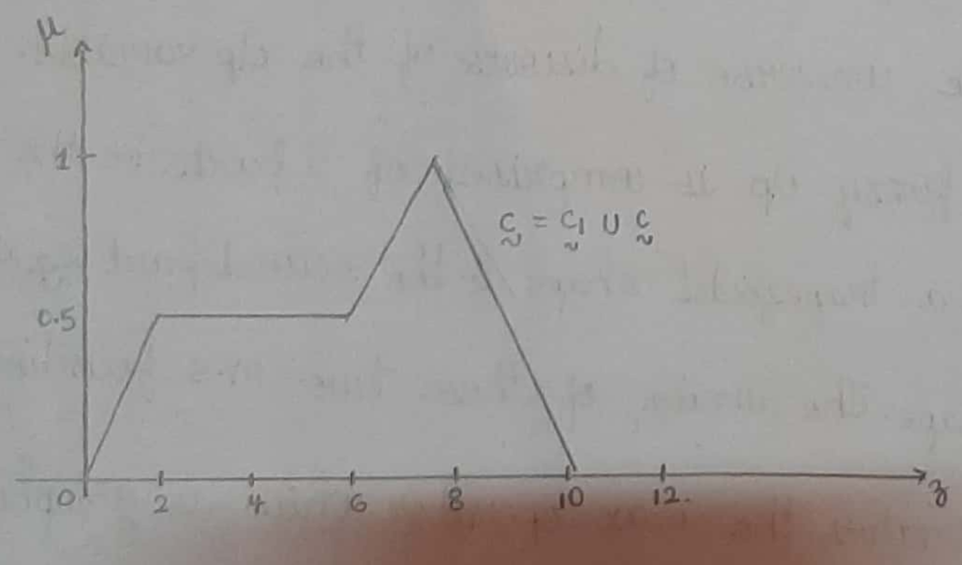
the outer envelope of the two shapes as shown in the fig below.



a) first part of fuzzy o/p, $\underset{\sim}{c_1}$



b) second part of fuzzy o/p, $\underset{\sim}{c_2}$



c) union of $\underset{\sim}{c_1}$ & $\underset{\sim}{c_2}$

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- The different defuzzification methods are as follows

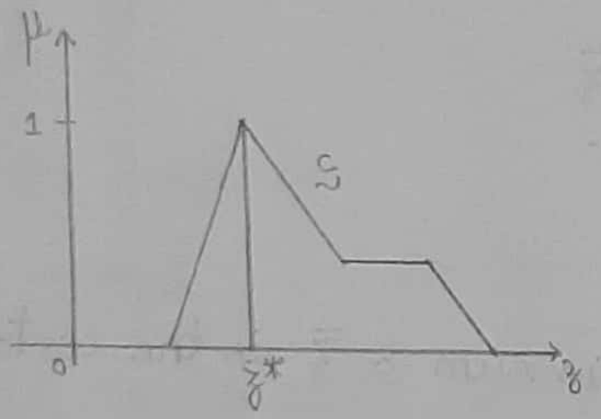
1. Max-membership defuzzification method
2. Centroid method
3. Weighted average method
4. Mean - Max membership method
5. centre of sums.

1. MAX-MEMBERSHIP DEFUZZICATION METHOD (Height Method):

- This method is given by the algebraic expression $\mu_{\underset{\sim}{C}}(z^*) \geq \mu_{\underset{\sim}{C}}(z)$

$$\mu_{\underset{\sim}{C}}(z^*) \geq \mu_{\underset{\sim}{C}}(z) \quad ; \quad \forall \; z \in Z$$

- It can be shown in the form of Venn dig as follows.



$z^* \rightarrow$ defuzzified value

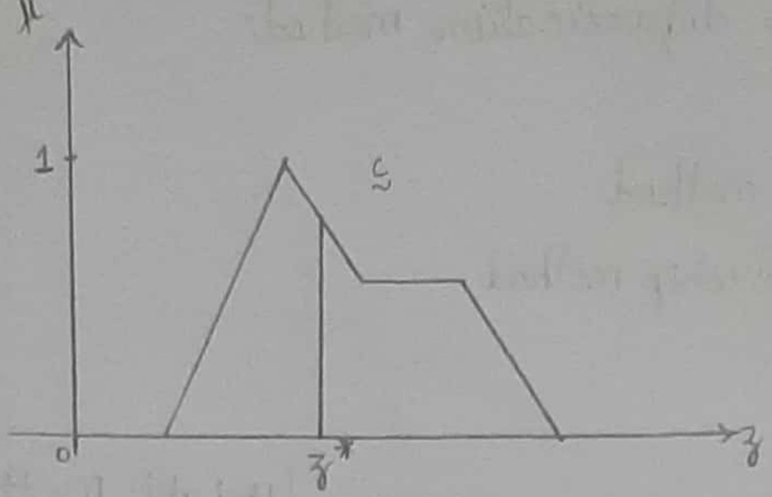
2. CENTROID DEFUZZIFICATION METHOD: (Centre of Area (or) Centre of gravity)

- It is given by algebraic expression

$$z^* = \frac{\int \mu_{\underset{\sim}{C}}(z) \cdot z \cdot dz}{\int \mu_{\underset{\sim}{C}}(z)\, dz}$$

= crisp value.

where "$\int$" denotes algebraic integral

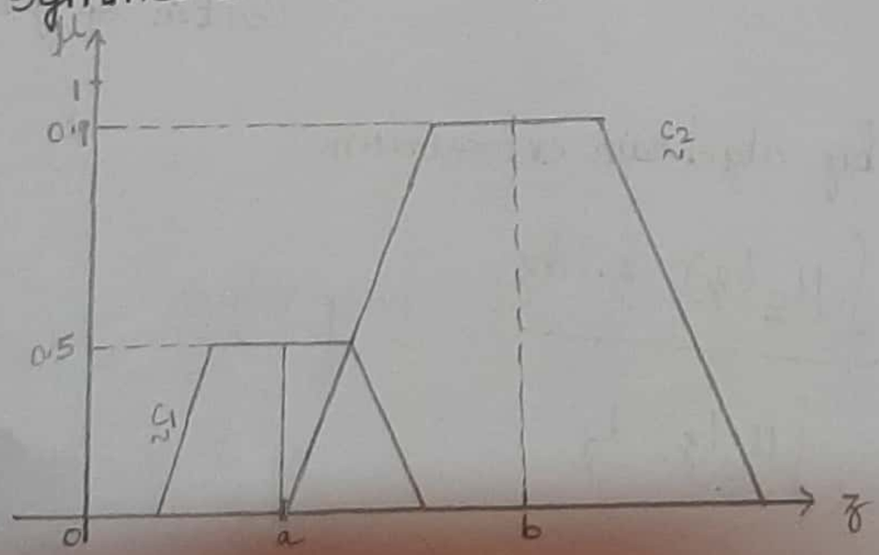


3. WEIGHTED AVERAGE METHOD-

- This method is valid for symmetrical o/p membership functions, it is given by mathematical expression

$$z^* = \frac{\sum \mu_{\underset{\sim}{C}}(\bar{z}) \cdot \bar{z}}{\sum \mu_{\underset{\sim}{C}}(\bar{z})}$$

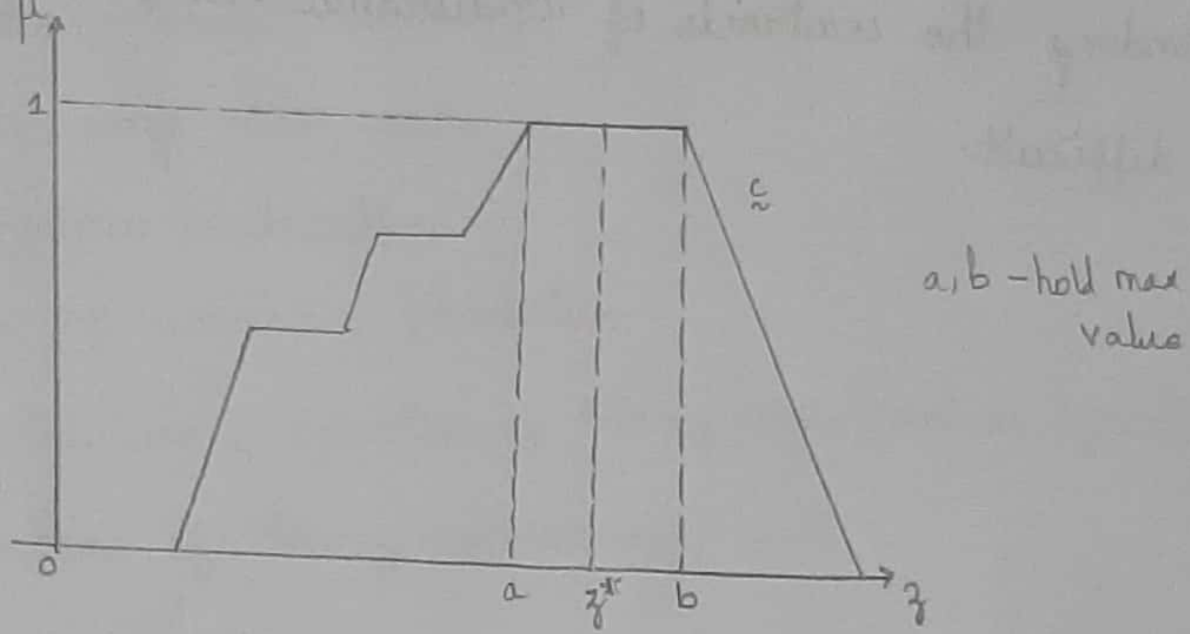
where 'Σ' denotes algebraic sum & $\bar{z}$ is the centroid of each symmetric membership function



4. MEAN - MAX MEMBERSHIP METHOD: (Middle of ~~Mean~~ Maxima method)

- It is given by $z^* = \frac{a+b}{2}$

where a & b are the points as shown in the fig given below.



5. CENTRE OF SUMS:

- This method is faster than many defuzzification method & not restricted to the symmetric m.s funct.
- This involves algebraic sum of individual o/p fuzzy sets $\underset{\sim}{C_1}$ & $\underset{\sim}{C_2}$
- It is mathematically expressed as follows

$$z^* = \frac{\sum_{K=1}^{n} \mu_{\underset{\sim}{C}K}(z) \cdot \int_{z} \bar{z} \cdot dz}{\sum_{K=1}^{n} \mu_{\underset{\sim}{C}K}(z) \cdot \int_{z} dz}$$

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